PHYSICS OF MATERIALS

Physics School Autumn 2025

Series 11 Solution **5 December 2025**

Exercise 1: Eutectic phase diagram

The phase diagram is expressed in terms of weight. The law of levers gives us the following:

$$\frac{m_\alpha}{m_\alpha + m_\beta} = \frac{X_\beta - X}{X_\beta - X_\alpha}$$
$$\frac{m_\beta}{m_\alpha + m_\beta} = \frac{X - X_\alpha}{X_\beta - X_\alpha}$$

At a temperature above the eutectic, we have the maximum solid phase in the liquid before solidification, which is given by:

$$\frac{m_\beta}{m_L + m_\beta} = \frac{80-60}{90-60} = 67\%$$ of pro-eutectic β_2 solid phase. Its concentration is 90% of the B element. The liquid has a eutectic concentration of 60%.

Right after the eutectic temperature T_3, the liquid (which has the composition of the eutectic L2 with 60% B) solidifies in two phases. Their weight ratio is given by:

$$\frac{m_\alpha}{m_\alpha + m_\beta} = \frac{90-60}{90-30} = 50\%$$
$$\frac{m_\beta}{m_\alpha + m_\beta} = \frac{60-30}{90-30} = 50\%$$

In the end, we have the following:

$67\% + 0.5 \cdot 33\% = 83\%$ of β phase and 17% of α phase.

The calculations at equilibrium for the temperature T_3 give a decomposition into phases $\alpha(30\%B) + \beta(90\%B)$ of which the weight ratios are:

$$\frac{m_\alpha}{m_\alpha + m_\beta} = \frac{90-80}{90-30} = 17\%$$

$$\frac{m_\beta}{m_\alpha + m_\beta} = \frac{80-30}{90-30} = 83\%$$

Exercise 2 Iron-Carbon phase diagram

The eutectoid decomposition gives two phases: the α-ferrite and the cementite Fe_3C. The composition of the eutectoid is given by:

$$m_\alpha = \frac{X_{Fe_3C} - X}{X_{Fe_3C} - X_\alpha}(1\,kg) = \frac{6.69-0.77}{6.69-0.02} = 885\,g$$

$$m_{Fe_3C} = \frac{X - X_\alpha}{X_{Fe_3C} - X_\alpha}(1\,kg) = \frac{0.77-0.02}{6.69-0.02} = 115\,g$$

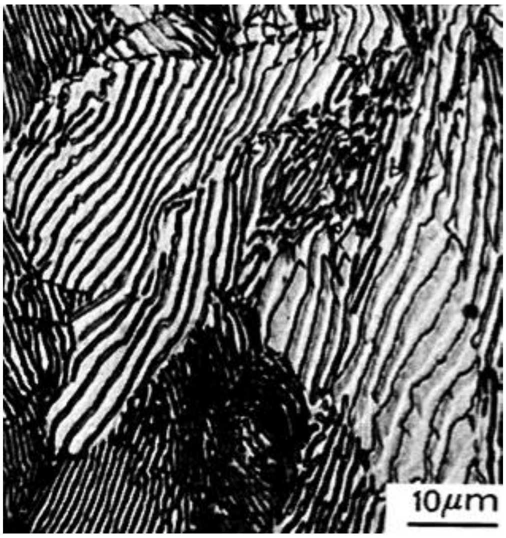

Figure 11-1: The structure of the steel's eutectoid is lamellar (like eutectic phases). We call this phase pearlite. This phase appears in an optical microscope as a dark color and pearl reflections (optical diffraction) from which its name derives.

When hypo-eutectoid steel (C<0.5wt%) is cooled, pro-eutectoid ferrite phase forms (see Fig. 11-2), and we can calculate its quantity with the phase diagram at equilibrium for 728°C.

$$m_\alpha = \frac{X_\gamma - X}{X_\gamma - X_\alpha}(1\,kg) = \frac{0.77-0.5}{0.77-0.02} = 360\,g$$

The remaining austenite (640 g) decomposes at 727°C, creating pearlite with the weight fractions calculated previously for the eutectoid composition. The equilibrium calculation yields the final partitioning between ferrite and cementite. All of the cementite concentration is contained within the pearlitic structure.

$$m_\alpha = \frac{X_{Fe_3C} - X}{X_{Fe_3C} - X_\alpha}(1\,kg) = \frac{6.69 - 0.5}{6.69 - 0.02} = 930\ g$$

$$m_{Fe_3C} = \frac{X - X_\alpha}{X_{Fe_3C} - X_\alpha}(1\,kg) = \frac{0.5 - 0.02}{6.69 - 0.02} = 70\ g$$

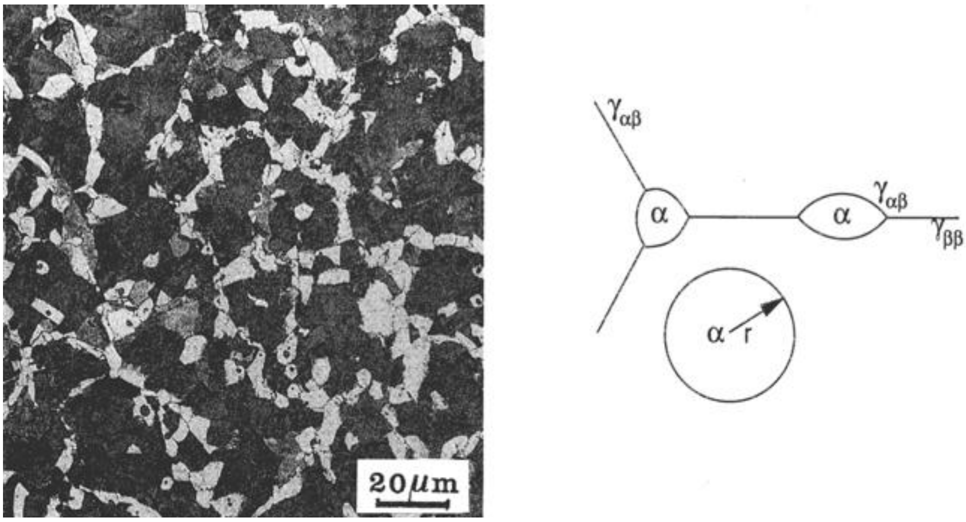

Fig. 11. 2 Formation mechanism of the pro-eutectoid ferrite (light colored) in the austenite. In the figure on the right, the dark phase is the pearlite arising from the austenite decomposition.

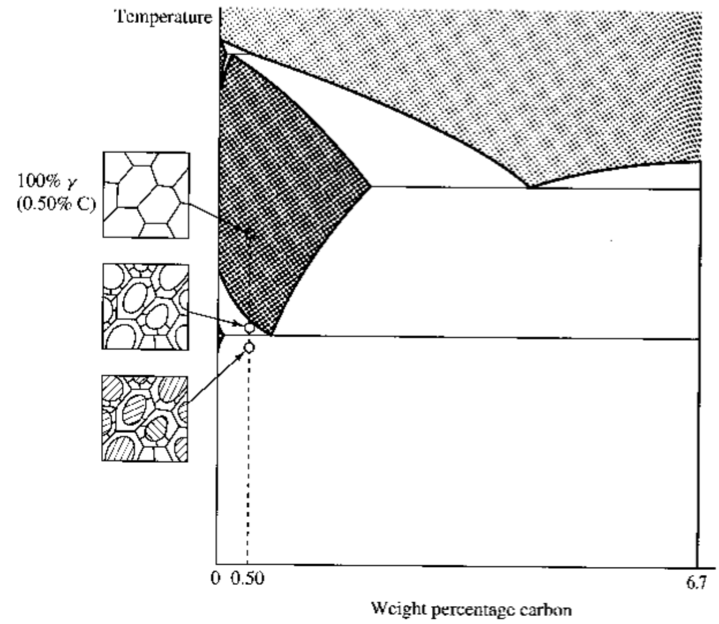

Fig. 11-3 Development of the microstructure of hypo-eutectoid steel.